

# C.U.SHAH UNIVERSITY

## Winter Examination-2015

**Subject Name: Engineering Mathematics-II****Subject Code: 4TE02EMT1****Semester: II****Time: 10:30 To 1:30****Branch: B.Tech(All)****Date: 19/11/2015****Marks: 70**

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1 Attempt the following questions: (14)**

- a) A square matrix  $A$  is called orthogonal if  
(a)  $AA^{-1} = I$       (b)  $A^2 = A$       (c)  $A^T = A^{-1}$       (d)  $A^2 = I$
- b) A  $n \times n$  Non-Homogeneous system of equations  $AX = B$  is given. If  $\rho(A) = \rho(A : B) = n$  then the system has  
(a) No solutions      (b) Unique solutions  
(c) Infinite solution      (d) None of these
- c) The rank of the matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is  
(a) 1      (b) 2      (c) 3      (d) -2
- d) The Sum of the eigenvalues of  $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  is  
(a) 1      (b) 4      (c) 2      (d) 5
- e) Find the value of  $\begin{vmatrix} 1 & 2 & 3 \\ 0 & -2 & 3 \\ 1 & 0 & 0 \end{vmatrix} = \underline{\hspace{2cm}}$   
(a) 1      (b) 12      (c) -2      (d) 0
- f) A square matrix  $A$  is called Singular if  
(a)  $|A| = 0$       (b)  $A^2 = A$       (c)  $AA^T = I$       (d)  $|A| \neq 0$
- g)  $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx = \underline{\hspace{2cm}}$   
(a) 0      (b) 1      (c)  $\frac{\pi}{2}$       (d)  $\frac{1}{2}$



- h)  $\int_0^{\pi/2} \cos^4 x \, dx = \underline{\hspace{2cm}}$   
 (a) 0      (b) 1      (c)  $\frac{3\pi}{16}$       (d)  $\frac{8\pi}{3}$
- i)  $\int_0^1 \int_0^x dy \, dx = \underline{\hspace{2cm}}$   
 (a)  $\frac{1}{2}$       (b) -1      (c) 0      (d) y
- j) The value of  $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$  for  $m \neq \pm n$  is  
 (a) 0      (b)  $\pi$       (c)  $\frac{\pi}{2}$       (d)  $2\pi$
- k) Angle between the vectors  $2i + 2j - k$  and  $6i - 3j + 2k$  is  
 (a)  $\cos^{-1}\left(\frac{4}{11}\right)$       (b)  $\cos^{-1}\left(\frac{4}{21}\right)$       (c)  $\sin^{-1}\left(\frac{4}{11}\right)$       (d)  $\cos^{-1}\left(\frac{4}{21}\right)$
- l)  $\text{div curl } \vec{V} = \underline{\hspace{2cm}}$   
 (a) 0      (b) 1      (c)  $\vec{0}$       (d)  $\vec{V}$
- m) A vector  $\vec{F}$  is said to be irrotational if  
 (a)  $\nabla \times \vec{F} = 0$       (b)  $\nabla \cdot \vec{F} = 0$       (c)  $\nabla \vec{F} = 0$       (d) None of these
- n) If  $\begin{bmatrix} x & 2 \\ 3 & 1 \end{bmatrix}$  is a singular matrix then  $x = \underline{\hspace{2cm}}$   
 (a) 1      (b) 6      (c) 2      (d) -6

Attempt any four questions from Q-2 to Q-8

**Q-2 Attempt all questions**

a) Find the inverse of the matrix  $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & 1 \end{bmatrix}$  by using determinant method. (05)

b) Evaluate:  $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$  (05)

c) Reduce the matrix  $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$  to the normal form and find its rank. (04)



**Q-3 Attempt all questions**

a) Find the eigenvalues & eigenvectors of a matrix  $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$  (05)

b) Solve the following system of equations by Cramer's rule: (05)  
 $x + 2y - z = 3; \quad x + y + 2z = 9; \quad 2x + y - z = 2$

c) Determine  $\int_0^1 \ln x \, dx$  converge or diverges. (04)

**Q-4 Attempt all questions**

a) Find the volume common to the cylinder  $x^2 + y^2 = a^2$  and  $x^2 + z^2 = a^2$ . (05)

b) Find the inverse of the following matrix by using elementary transformation (05)

$$A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \\ 2 & 1 & 2 & 1 \\ 3 & -2 & 1 & 6 \end{bmatrix}$$

c) Solve:  $\frac{dy}{dx} + y \tan x = \sin 2x, \quad y(0) = 1$  (04)

**Q-5 Attempt all questions**

a) Obtain Row echelon & Reduced row echelon form of the following matrix: (05)

$$A = \begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$$

b) Solve:  $\frac{dy}{dx} = 2y \tan x + y^2 \tan^2 x$  (05)

c) Find the directional derivatives of  $\phi = xy^2 + yz^2$  at the point  $(2, -1, 1)$  in the direction of the vector  $\hat{i} + 2\hat{j} + 2\hat{k}$ . (04)

**Q-6 Attempt all questions**

a) Evaluate  $\int_C \vec{F} \cdot d\vec{r}$  where  $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$  and  $C$  is the rectangle in the  $xy$ -plane bounded by  $y = 0, x = a, y = b, x = 0$ . (05)



b) Evaluate  $\iint_S \vec{F} \cdot \hat{n} ds$ , where  $\vec{F} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$  and S is the part of the plane **(05)**

$2x + 3y + 6z = 12$  in the first octant.

c) Solve the system of equation by Gauss-Elimination method. **(04)**

$$2x + 2y + 2z = 0$$

$$-2x + 5y + 2z = 1$$

$$8x + y + 4z = -1$$

**Q-7 Attempt all questions**

a) Change the order of integration and evaluate  $\int_0^a \int_{a-\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} dx dy$ . **(05)**

b) Solve:  $\left(x + \frac{ay}{x^2 + y^2}\right)dx + \left(y - \frac{ax}{x^2 + y^2}\right)dy = 0$  **(05)**

c) Evaluate:  $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$  **(04)**

**Q-8 Attempt all questions**

a) Verify Green's theorem for  $\oint_C [(x - y)dx + 3xy dy]$  where C is the boundary **(07)**

of the region bounded by the parabolas  $x^2 = 4y$  and  $y^2 = 4x$ .

b) State Cayley-Hamilton theorem and Find the characteristic equation for the **(07)**

matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ . Also find the matrix represented

by  $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ .

